Title: Lamb Wave Decomposition Using Amplitude Matching with Concentric Circular PZT Transducers

Authors: *Chulmin Yeum*[1]*, Hoon Sohn, Jeong Beom Ihn*[2]

[1] The Department of Civil and Environmental Engineering, Korea Advanced Institute of Science and Technology, Daejeon 305-701, South Korea

[2] Boeing Phantom Works, Advanced Structures Technology, PO Box 3707, MC 45-13 Seattle, WA 98124-2207

ABSTRACT

Lamb waves using surface-bonded piezoelectric transducers (PZTs) have been widely applied to Non Destructive Evaluation (NDE). However, the identification of individual Lamb wave modes and the subsequent data interpretation are often difficulty due to the dispersive and multimodal natures of Lamb waves. To tackle this problem, several techniques to isolate a specific Lamb wave mode of interest have been proposed previously. They include (1) a technique that requires placement of collocated PZT transducers on both surfaces of a specimen, (2) one based on the tuning of the driving frequency with respect to PZT size and the plate thickness, and (3) an array of PZT transducers with time delays, to name a few. In this study, a Lamb wave decomposition technique using newly designed concentric circular PZT transducers attached on a single side of a plate is proposed. The advantages of this approach compared to the previous approaches are that (1) PZT transducers need to be placed only a single surface of a specimen and (2) mode decomposition can be performed at any desired frequency band. The proposed mode decomposition technique is formulated by solving 3D Lamb wave propagation equations considering the PZT size and shape effects. The effectiveness of the proposed method is investigated through numerical simulations, and experimental tests performed on an aluminum plate.

INTRODUCTION

For structural health monitoring (SHM) and nondestructive testing (NDT) of plate-like structures, Lamb waves have been gained a great deal of attention to monitoring and maintenance of aerospace, civil infrastructural and mechanical systems. Many researchers have developed Lamb wave based SHM on various materials and applications. In particular, researchers have recognized the potential use of piezoelectric transducers (PZTs) for Lamb waves based SHM [1]-[3].

The analysis and interpretation of Lamb waves can be complicated due to their dispersive and multimodal natures. The various frequency components of Lamb waves travel at different speeds and the shapes of wave packets changes as they propagate though a solid medium. Thus interaction of waves with a defect can result in a complicated multimode signal. In order to make signal interpretation and processing of Lamb wave signals easier, any desired Lamb wave mode should be extracted from a multimode signal.

To eliminate these complications, this proposed technique allows extracting any desired fundamental Lamb wave modes (S_0 and A_0 modes) at any driving frequency without any changing of PZT size and/or spacing configuration. Additionally, the mode decomposition technique can be omnidirectionally applied without any preferred direction of wave propagation. For this purpose, a new PZT design of combining concentric disk and ring PZTs called a dual PZT is used.

THEORETICAL FORMULATION

Asymptotic solution for a response model of surface bonded circular PZTs

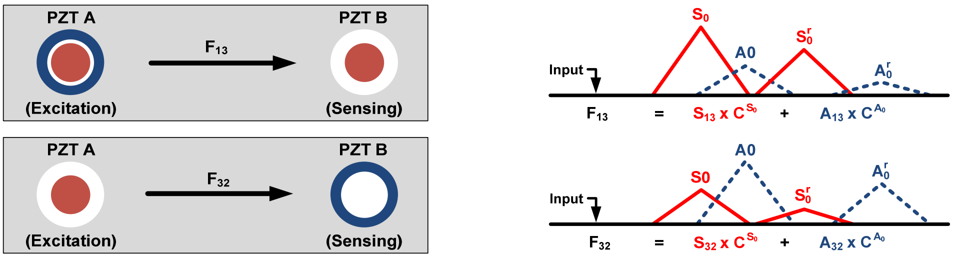
Consider the isotropic plate with one pair of exciting and sensing circular PZTs bonded on the top surface. Based on the Lamb wave propagation model between circular exciting and sensing PZTs, the asymptotic solution for the output voltage of the circular sensing PZT can be obtained:

$$v(t) = -i\frac{2\sqrt{2\pi}\tau_0 E_s h_s g_{31} a}{\mu c\sqrt{r_s}}\left[\begin{array}{l}\dfrac{1}{\sqrt{\xi^{S_0}}}J_1\left(\xi^{S_0}a\right)J_1\left(\xi^{S_0}c\right)\dfrac{N_S\left(\xi^{S_0}\right)}{D_S^{'}\left(\xi^{S_0}\right)}e^{i(\varpi t-\frac{\pi}{4}+\xi^{S_0}r_s)} \\ +\dfrac{1}{\sqrt{\xi^{A_0}}}J_1\left(\xi^{A_0}a\right)J_1\left(\xi^{A_0}c\right)\dfrac{N_S\left(\xi^{A_0}\right)}{D_S^{'}\left(\xi^{A_0}\right)}e^{i(\varpi t-\frac{\pi}{4}+\xi^{A_0}r_s)}\end{array}\right] \quad (1)$$

where a and c are the radius of actuator and sensor. λ and μ are Lame constants for the plate material, and ρ is the material density of the plate. E_S, h_S and g_{31} are Young's modulus, thickness, and xz-directional piezoelectric voltage constant of the sensing PZT. The wave number ξ of a specific mode for a given ω is obtained from the solutions of the Rayleigh-Lamb equation in an isotropic plate. Ns and Ds are defined in an earlier study [3]-[4].

Formulation of Lamb wave decomposition technique using circular PZTs

To achieve the proposed mode decomposition concept, different size of PZTs should be placed at the identical position and also at least two combinations of actuator and sensor are needed. To address this issue, a dual PZT which is a single PZT unit by combining concentric disk and ring type PZTs is introduced.



(a) (b)

Figure 1. A basic concept of the proposed Lamb wave decomposition technique : (a) The configuration of Lamb wave signals generated and sensed by dual PZTs: F_{13} and F_{32} denote the response signals at the disk and ring parts of PZT *B* when whole and disk parts of PZT *A* is used for excitation, respectively. (b) A schematic diagram of S_0 and A_0 mode signals included in F_{13} and F_{32} signals: S_0 and A_0 mode signals are expressed as the product of common functions (C^{S_0} and C^{A_0}) and scaling factor *S* and *A* with subscription *ij* (superscript *r* denotes reflection wave from structure boundaries)

A key concept of the proposed Lamb wave decomposition technique is presented in figure 1. Figure 1(a) shows three PZTs placed on the top surface of the plate. Notations for output signals are defined as following; (1) '*F*' denotes measured a forward signal (2) '*i*' and '*j*' indicate actuator and sensor parts (3) '*1*', '*2*' and '*3*' denote whole, ring and disk parts. Based on the asymptotic solution for the response model derived in the previous section, exact output responses are as follow:

$$F_{ij} = S_{ij}C^{S_0} + A_{ij}C^{A_0} \tag{2}$$

where $S_{33} = E_3^{S_0} M_3^{S_0}$; $S_{23} = (E_1^{S_0} - E_2^{S_0}) M_3^{S_0}$; $S_{13} = S_{23} + S_{33}$

$$S_{32} = E_3^{S_0}(A_1 - A_2)^{-1}(A_1 M_1^{S_0} - A_2 M_2^{S_0})$$

$$S_{22} = (E_1^{S_0} - E_2^{S_0})(A_1 - A_2)^{-1}(A_1 M_1^{S_0} - A_2 M_2^{S_0}),$$

$$S_{21} = (A_1 - A_2 + A_3)^{-1}\{(A_1 - A_2)S_{22} + A_3 S_{23}\} \quad ; \quad S_{12} = S_{22} + S_{32}$$

$$S_{31} = (A_1 - A_2 + A_3)^{-1}\{(A_1 - A_2)S_{32} + A_3 S_{33}\} \quad ; \quad S_{11} = S_{21} + S_{31}$$

$$E_i^{S_0} = a_i J_1\left(\xi^{S_0} a_i\right) \quad ; \quad M_j^{S_0} = \frac{1}{a_j} J_1\left(\xi^{S_0} a_j\right) \quad ; \quad A_j = \pi (a_j)^2$$

$$C^{S_0} = -i \frac{2\sqrt{2\pi}\tau_0 E_s h_s g_{31}}{\mu\sqrt{r_s}} \frac{N_S\left(\xi^{S_0}\right)}{D_S'\left(\xi^{S_0}\right)} e^{i(\omega t - \frac{\pi}{4} + \xi^{S_0} r_s)}$$

A_{ij} are defined in a similar fashion.

Here, C^{S_0} represents a common function for S_0 mode which is independent of actuator and sensor size. The functions of actuator and sensor size for S_0 mode are defined as $E_i^{S_0}$ and $M_j^{S_0}$, respectively. Note that notations for A_0 mode are similar to that of S_0 mode. *S* and *A* with subscript *ij* are defined as scaling factors of S_0 and A_0 modes included F_{ij} signals.

$$\begin{bmatrix} F_{11} \\ F_{12} \\ \vdots \\ F_{33} \end{bmatrix} = \begin{bmatrix} S_{11} & A_{11} \\ S_{12} & A_{12} \\ \vdots & \vdots \\ S_{33} & A_{33} \end{bmatrix} \begin{bmatrix} C^{S_0} \\ C^{A_0} \end{bmatrix} \tag{3}$$

Based on Eq. (2), F_{ij} signals can be represented by the matrix formulation in Eq. (3). Since E and M functions always produce real values at varying the size of actuator and sensor, S_0 or A_0 mode signals included in F_{ij} signals have identical signal shape and arrival time but different amplitude. As shown in figure 3(b), amplitudes of S_0 and A_0 mode signals are varied differently as the size of the exciting or sensor PZT is changed.

The procedure of the proposed technique is as follow: First, the extraction of the common functions is achieved by taking the pseudo inverse of the scaling factor matrix and premultiply to the signal matrix. Second, using the derived common functions, decomposition of S_0 and A_0 modes in F_{ij} signals can be computed by the product of the corresponding scaling factors and common functions. For example, S_0 mode included in signals F_{32} can be computed by the product of S_{32} and C^{S_0} .

NUMERICAL SIMULATION

Numerical simulation setup

The concept of the proposed mode decomposition technique using dual PZTs was confirmed through numerical FEM simulation with a MSC/NASTRAN. A 3-D aluminum plate of 200 mm × 400 mm × 3 mm with three dual PZTs was modeled. Two dual PZTs are located at an identical position but different sides. A narrowband toneburst input traction force was applied to equally divided 8 points of the edge of the exciting PZT parts, respectively. The corresponding output strain field was measured through averaging strain of the sensing PZT area.

Numerical simulation result

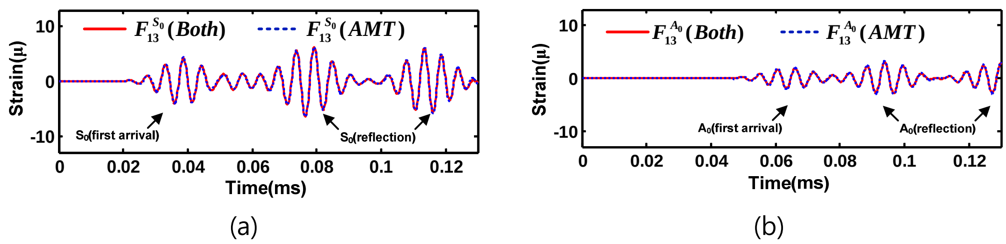


Figure 2. Comparison decomposed S_0 and A_0 mode signals using the collocated dual PZTs for selective generation denoted '*Both*' to the proposed amplitude matching technique denoted '*AMT*' : (a) S_0 mode (b) A_0 mode ('*Both*' indicates that a selective mode can be generated by using collocated dual PZTs. '*AMT*' means the signal decomposition based on the proposed amplitude matching

technique.)

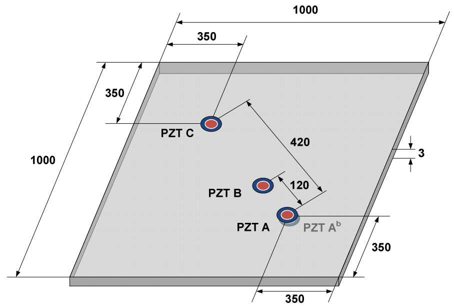


Figure 3. The configuration and size of the test specimen including four dual PZTs

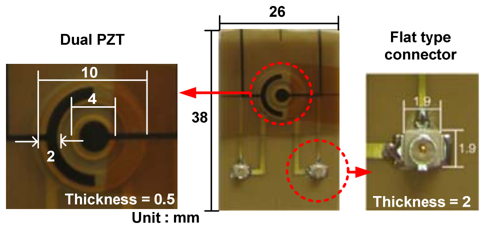


Figure 4. The dimension of the new dual PZT with flat type connectors used in this study

Figure 2 illustrates S_0 and A_0 mode signals obtained from F_{13} and F_{23} to establish the proposed amplitude matching technique. In figure 2, decomposed S_0 and A_0 mode signals in F_{ij} signals are represented by $F_{ij}^{S_0}$ and $F_{ij}^{A_0}$. As result of comparison of 'Both' and 'AMT', decomposed S_0 and A_0 mode signals based on the amplitude matching technique are pretty well matched with first arrival S_0 and A_0 mode signals as well as reflected ones.

TEST RESULTS

Experimental setup

The dimension of the aluminum plate used in this study were 1000mm x 1000mm x 3mm and three dual PZTs were mounted on the top of the plate and one is attached on other side of the plate shown in figure 3. A long path AC is used for calculating relative scaling factors. On the other hand, PZT B was closely installed with PZT A to obtained overlapping S_0 and A_0 mode signals to verify the proposed method. Figure 4 shows a new dual PZT with flat connectors used in this study [5].

Experimental results

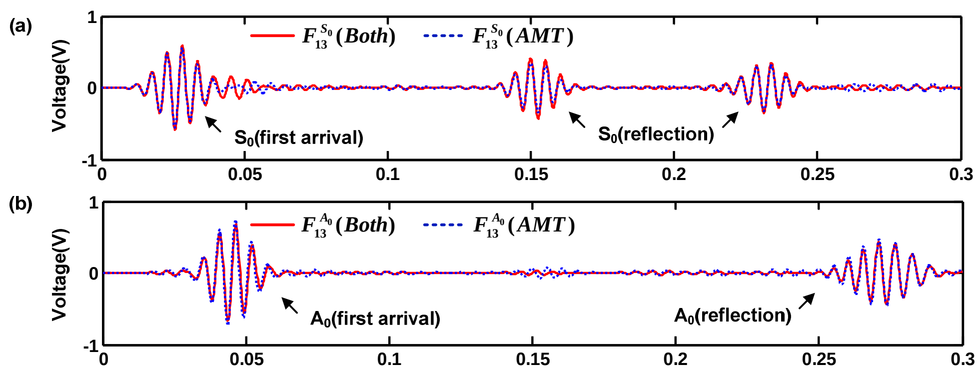


Figure 5. Comparison decomposed S_0 and A_0 mode signals using the collocated dual PZTs for selective generation denoted '*Both*' to the proposed amplitude

matching technique denoted '*AMT*': (a) S_0 mode (b) A_0 mode

The objective of the proposed technique is to decompose measured signals into S_0 and A_0 modes using relative scaling factors obtained from a long path where first arrival and reflected S_0 and A_0 mode signals are separated. In the experiment setup, S_0 and A_0 modes included a signal measured at the AB path will eventually be decomposed using relative scaling factors derived from the AC path.

Figure 5 illustrates S_0 and A_0 mode signals obtained from F_{13} and F_{23} to experimentally validate the proposed amplitude matching technique. As results, decomposed S_0 and A_0 mode signals were well matched with real mode signals from collocated dual PZTs. Even the first and second reflected S_0 or/and A_0 mode signals can be effectively extracted from measured signals.

CONCLUSION

In this study, the Lamb wave mode decomposition technique using single side attached dual PZTs were presented. The theoretical model for circular PZTs and dual PZTs was first derived. Then, the Lamb wave decomposition technique was conceived based on the amplitude difference of individual modes obtained from difference size of actuator and sensor PZTs. Numerical simulations and experimental tests were conducted to validate the effectiveness of the proposed technique. As results, it is expected that the individual mode signals can be effectively extracted from measured signals.

ACKNOWLEDGEMENTS

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